PH16212, Homework 5

Deadline: Nov. 18, 2019

1. Consider the two-loop massive sunset diagram with *equal mass*

$$D_1 = l_1^2 - m^2, \quad D_2 = l_2^2 - m^2, \quad D_3 = (l_2 + l_1 + p)^2 - m^2, \tag{1}$$

with $p^2 = s$. We define two irreducible scalar products as

$$D_4 = (l_1 + p)^2, \quad D_5 = (l_2 + p)^2 \tag{2}$$

As usual, we define the variables

$$y_{11} = l_1^2, \quad y_{22} = l_2^2, \quad y_{12} = l_1 \cdot l_2, \quad x_{11} = l_1 \cdot p, \quad x_{21} = l_2 \cdot p, \tag{3}$$

- Write down D_i's, $i = 1, \ldots 5$ as linear functions of x and y's.
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- Explicitly write down the Baikov polynomial P as a function of x and y's.
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2. Let l_1 and l_2 be two D-dimensional loop momenta, p_1, p_2, p_3 be the independent external momenta. Prove the identity

$$G\begin{pmatrix} l_1 & l_2 & p_1 & p_2 & p_4 \\ l_1 & l_2 & p_1 & p_2 & p_4 \end{pmatrix} = \frac{G\begin{pmatrix} l_1 & p_1 & p_2 & p_4 \\ l_1 & p_1 & p_2 & p_4 \end{pmatrix} G\begin{pmatrix} l_2 & p_1 & p_2 & p_4 \\ l_2 & p_1 & p_2 & p_4 \end{pmatrix} - G\begin{pmatrix} l_1 & p_1 & p_2 & p_4 \\ l_2 & p_1 & p_2 & p_4 \end{pmatrix}^2}{G\begin{pmatrix} p_1 & p_2 & p_4 \\ p_1 & p_2 & p_4 \end{pmatrix}} \tag{4}$$

This identity is very useful for two-loop integrand analysis.

3. Consider the two-loop massless pentagon-box diagram with the propagators,

$$\begin{gathered} D_1 = l_1^2, \quad D_2 = (l_1 - p_1)^2, \quad D_3 = (l_1 - p_1 - p_2)^2, \quad D_4 = (l_1 - p_1 - p_2 - p_3)^2 \\ D_5 = l_2^2, \quad D_6 = (l_2 - p_5)^2, \quad D_7 = (l_2 - p_4 - p_5)^2, \quad D_8 = (l_1 + l_2)^2 \end{gathered} \tag{5}$$

with $p_1^2 = p_2^2 = p_3^2 = p_4^2 = p_5^2 = 0$.

- Draw the corresponding Feynman diagram.

- Assume that l_1 and l_2 are 4-dimensional. Consider the parameterization,

$$
\begin{aligned}
l_1 &= (1+a_1)p_1 + a_2 p_2 + a_3 \frac{\langle 23\rangle}{\langle 13\rangle}\lambda_1\tilde{\lambda}_2 + a_4 \frac{[23]}{[13]}\lambda_2\tilde{\lambda}_1 \\
l_2 &= b_1 p_4 + (1+b_2)p_5 + b_3 \frac{\langle 51\rangle}{\langle 41\rangle}\lambda_4\tilde{\lambda}_5 + b_4 \frac{[51]}{[41]}\lambda_5\tilde{\lambda}_4
\end{aligned} \tag{6}
$$

Solve the maximal cut equation

$$
D_1 = D_2 = D_3 = D_4 = D_5 = D_6 = D_7 = D_8 = 0\,. \tag{7}
$$

How many solutions are there? For each solution, explicitly write down the expression for a and b's as *rational factions* of s_{12}, s_{23}, s_{34}, s_{45} and s_{15} and

$$
\mathrm{tr}_5 = 4i\,\det(p_1, p_2, p_3, p_4)\,. \tag{8}
$$

(Hint: use momentum twistor method.)